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**211. (April, 1914.) Proposed by E. T. BELL, University of Washington.**

If an odd perfect number exists, the total number of its divisors is a multiple of 2, but not of 4; or, what is the same thing, an odd perfect number must be of the form  $p^{2a-1}n^2$ , where  $p$  is prime and  $a$  is odd.

**214. (April, 1914.) Proposed by A. J. KEMPNER, University of Illinois.**

Let  $a$  be a positive integer  $\geq 2$ , and let  $T(n)$  denote the number of distinct divisors of the positive integer  $n$ , including both 1 and  $n$ , so that  $T(1) = 1$ ,  $T(2) = 2$ ,  $T(3) = 2$ ,  $T(4) = 3$ ,  $\dots$ . Show that

$$\sum_{n=1}^{n=\infty} T(n)/a^n = \sum_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case  $a = 10$  gives, as is easily seen:

$$9 \sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \dots$$

**217. (May, 1914.) Proposed by E. T. BELL, University of Washington.**

(i) If  $r$  is a prime greater than 2, and  $p \equiv 2^r + 1$  is prime, the only solution, when  $n$  is greater than 2, of  $x^n - y^n = p$ , is  $n = 3$ ,  $x = 2$ ,  $y = 1$ .

(ii) The only primes that are simultaneously of the forms  $4k + 1$  and  $3^m - 2^m$  are 1 and 5.

(iii) Generalize (ii).

**219. (June, 1914.) Proposed by E. D. CARMICHAEL, University of Illinois.**

Determine whether it is possible for a polygon to have the number of its diagonals equal to a perfect fourth power.

**221. (September, 1914.) Proposed by T. E. MASON, Purdue University.**

Find a number  $x$  such that the sum of the divisors of  $x$  is a perfect square. [Carmichael, *Theory of Numbers*, page 17.]

**222. (October, 1914.) Proposed by A. H. HOLMES, Brunswick, Me.**

Find rational values for  $m$  and  $n$  such that  $(m^2 + 1)/m^2 + (n^2 + 1)/n^2$  may be the square of an integer.

**223. (October, 1914.) Proposed by T. E. MASON, Purdue University.**

Show that

$$\frac{(rst)!}{t!(s!)^t(r!)^{st}}$$

is an integer,  $r$ ,  $s$ , and  $t$  being positive integers. Generalize to the case of  $n$  integers,  $r$ ,  $s$ ,  $t$ ,  $u$ ,  $\dots$ . [Carmichael, *Theory of Numbers*, page 28.]

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

**443. Proposed by A. M. KENYON, Purdue University.**

If  $p_r$  denote the sum of all the  $r$ -factor products that can be formed from the first  $n$  natural numbers ( $p_r = 0$  for  $r > n$ ), and if

$$D_s = \begin{vmatrix} p_1 & 1 & 0 & \dots & 0 \\ 2p_2 & p_1 & 1 & \dots & 0 \\ 3p_3 & p_2 & p_1 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ sp_s & p_{s-1} & p_{s-2} & \dots & p_1 \end{vmatrix}$$

show that

$$\sum_{i=0}^k (-1)^i c_i \binom{k}{i} D_{2k-i} = 0, \quad k, n = 1, 2, 3, \dots,$$

where

$$c_i = \frac{2k+1-i}{1+i}$$

when  $i$  is even and  $(2n+1)$  when  $i$  is odd; and  $\binom{k}{i}$  is the coefficient of  $x^i$  in  $(1+x)^k$ .

### SOLUTION BY THE PROPOSER.

The roots of the equation

$$x^n - p_1 x^{n-1} + p_2 x^{n-2} - \dots + (-1)^n p_n = 0$$

are the natural numbers  $1, 2, 3, \dots, n$ .

Solving Newton's formulæ<sup>1</sup> for the sums of like powers of the roots, we obtain

$$1^s + 2^s + 3^s + \dots + n^s = D_s, \quad n, s = 1, 2, 3, \dots$$

A relation between the  $D$ 's of odd subscript has been published<sup>2</sup> which is equivalent to

$$(1) \quad \sum_{i=0}^{I(k/2)} \binom{k}{2i+1} D_{2k-1-2i} = 2^{k-1} D_1^k, \quad k, n = 1, 2, 3, \dots,$$

and the following relation<sup>3</sup> exists among the  $D$ 's of even subscript,

$$(2) \quad \sum_{i=0}^{I(k/2)} \frac{2k+1-2i}{1+2i} \binom{k}{2i} D_{2k-2i} = (2n+1) 2^{k-1} D_1^k, \quad k, n = 1, 2, 3, \dots$$

These formulæ, in which  $I(k/2)$  denotes the integral part of  $k/2$ , are readily established by induction. Multiplying (1) by  $2n+1$  and subtracting the result from (2) we get the formula sought.

#### 444. Proposed by J. E. ROWE, Pennsylvania State College.

Prove that the determinant

$$\begin{vmatrix} \cot A, & \cot B, & \cot C \\ 1, & 1, & 1 \\ \cos^2 A, & \cos^2 B, & \cos^2 C \end{vmatrix} = 0, \quad \text{if} \quad A + B + C = 180^\circ.$$

### SOLUTION BY S. E. RASOR, Ohio State University.

Transforming trigonometrically and rearranging, the determinant becomes

$$\frac{-1}{4 \sin A \sin B \sin C} \begin{vmatrix} 2 \cos A \sin B \sin C, & 2 \cos B \sin C \sin A, & 2 \cos C \sin A \sin B \\ 1, & 1, & 1 \\ 2 \sin^2 A, & 2 \sin^2 B, & 2 \sin^2 C \end{vmatrix}.$$

By the formula  $2 \sin A \sin B \cos C = \sin^2 A + \sin^2 B - \sin^2 C$  when  $A + B + C = 180^\circ$ , this reduces to

$$\frac{-1}{4 \sin A \sin B \sin C} \begin{vmatrix} \sin^2 B + \sin^2 C - \sin^2 A, & \sin^2 A + \sin^2 C - \sin^2 B, & \sin^2 A + \sin^2 B - \sin^2 C \\ 1, & 1, & 1 \\ 2 \sin^2 A, & 2 \sin^2 B, & 2 \sin^2 C \end{vmatrix},$$

<sup>1</sup> See, for example, *Cajori's Theory of Equations*, pages 85-86.

<sup>2</sup> Stern, *Crelle's Journal*, volume 84, pages 216-218.

<sup>3</sup> *Proceedings Indiana Academy of Sciences*, 1914, page 440.